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Seismic response and damage analysis of buildings supported on flexible soils

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SUMMARY

The investigation reported in this paper studies the effects of soil–structure interaction (SSI) on the seismic response and damage of building–foundation systems. A simple structural model is used for conducting a parametric study using a typical record obtained in the soft soil area of Mexico City during the 1985 earthquake. Peak response parameters chosen for this study were the roof displacement relative to the base and the hysteretic energy dissipated by the simple structural model. A damage parameter is also evaluated for investigating the SSI effects on the seismic damage of buildings. The results indicate that in most cases of inelastic response, SSI effects can be evaluated considering the rigid-base case and the SSI period. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: seismic response; damage analysis; soil–structure interaction; buildings

1. INTRODUCTION

The investigation reported in this paper studies the effects of soil–structure interaction (SSI) on the inelastic seismic response of building–foundation systems. Emphasis is made on the case of Mexico City where extensive damage was observed in buildings on soft soil during the September 19, 1985 earthquake. The objectives of this study were to evaluate the importance of SSI effects on the seismic response and damage of buildings in Mexico City during the 1985 earthquake and to compare the results of this evaluation to those assuming an ideal rigid foundation. A basic approach in this study is to use a simple model for analysing the overall seismic behaviour of multistorey building structures. The simple model responds as a SDOF in its fixed-base condition. A parametric study is conducted using the EW component of the ground acceleration recorded at the SCT station in Mexico City during the 1985 earthquake.

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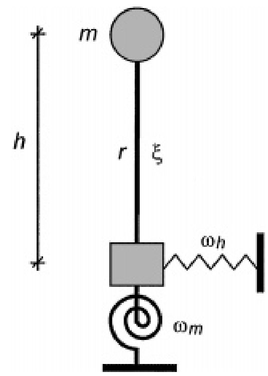

Figure 1. Single-storey structure on flexible soil.

2. DESCRIPTION OF THE MODEL AND METHOD OF ANALYSIS

2.1. Single-storey structure on flexible soil

The single-storey structure of height h on flexible soil shown in Figure 1 was considered in this study. It consists of a nonlinear structure of mass m resting on deformable soil. This structure responds as a SDOF system with a circular frequency ω in its fixed-base condition. According to some results in the literature [1], the mass at the base and the centroidal moment of inertia of the top mass can be neglected in an approximate analysis of the seismic response of simple non-linear building–foundation systems. Hence, the equations of motion of the structure–foundation model shown in Figure 1 in coupled horizontal translation and rocking can be written as follows:

$$\ddot{u} + \ddot{v}_{\mathrm{g}} + \ddot{v}_0 + h\ddot{\theta} + 2\xi\omega\dot{u} + \frac{r}{m} = 0 \tag{1}$$

$$\ddot{u} + \ddot{v}_{\mathrm{g}} + \ddot{v}_0 + h\ddot{\theta} + \omega_h^2 v_0 = 0 \tag{2}$$

$$\ddot{u} + \ddot{v}_{\mathrm{g}} + \ddot{v}_0 + h\ddot{\theta} + \omega_m^2 h\theta = 0 \tag{3}$$

In these equations, u is the horizontal displacement of the top mass relative to the base, which results from the deformation of the superstructure; r is the resistance function of the structure and ξ is the fraction of critical structural damping; v_0 is the translation of the base mass in addition to the free field motion; v_{g} is the free-field horizontal ground displacement; and θ is the rotation of the base mass (see Figure 2). In the derivation of Equations (1)–(3), the soil–foundation system is assumed to respond in the elastic range. The parameters ω_h and ω_m are characteristic frequencies which depend on the translational and rotational stiffness of the soil–foundation system, respectively, and on the mass of the superstructure. In this study, the stiffness and damping associated

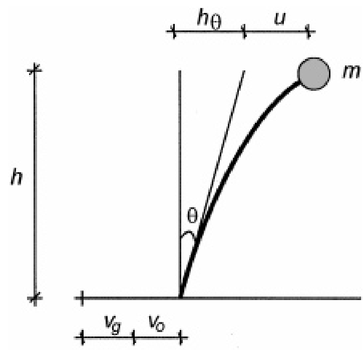

Figure 2. Components of displacements for the single-storey structure on flexible soil.

with the soil are assumed to be frequency-independent. For most buildings, the hypothesis of frequency-independent soil parameters leads to results with sufficient accuracy [2, 3].

The foundation damping, which is introduced by the contribution of radiation and material damping, also contributes to the overall damping of a building–foundation system. According to the approach adopted by the ATC3-06 [4], the effective damping factor of the building–foundation system, $\bar{\xi}$, is given by

$$\bar{\xi} = \bar{\xi}_0 + \frac{\xi}{(\bar{T}/T)^3} \tag{4}$$

where $\bar{\xi}_0$ represents the contribution of foundation damping, including both radiation and material damping. The second term on the right-hand side of Equation (4) represents the contribution of structural damping and depends on ξ and on the ratio $\bar{T}/T$, where $\bar{T}$ is the SSI fundamental period and T is the fixed-base fundamental period. The idea of defining an effective damping of the building–foundation system as the sum of a term that is proportional to the damping in the superstructure plus that from the foundation damping was first proposed by Bielak [5]. The same result was arrived at independently by Meek and Veletsos [6]. These ideas of effective damping were later elaborated by Jennings and Bielak [7], and by Veletsos and Meek [8].

Several aspects involved in obtaining a reliable estimation of the effective damping factor $\bar{\xi}$ are discussed in the following. One aspect is related to the radiation damping. According to Meek and Wolf [9], some soils have a cutoff frequency, below which there is no radiation damping. Another aspect to consider is that the foundation damping also depends on the slenderness ratio of a structure, which is not considered in this study. For instance, for the taller structures the contribution of radiation damping is generally small [10]. It is also of interest to mention that according to Equation (4), the flexibility of the building-foundation system reduces the effectiveness of the structural damping factor ξ, especially for high ratios of $\bar{T}/T$. A practical recommendation, that intents to consider such aspects involved in estimating the overall damping

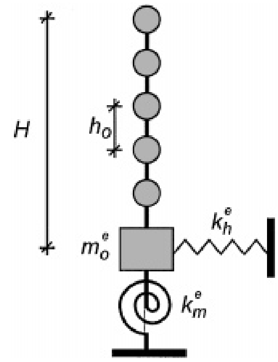

Figure 3. Idealized building–foundation system.

of a building-foundation system, is given by Veletsos [10]. This author recommends a minimum value for the overall damping factor of the building–foundation system, $\bar{\xi}$, which would be appropriate for a practical and simple approach to the problem. According to this recommendation, the value of $\bar{\xi}$ should never be taken less than the estimated value of the structural damping factor ξ. This recommendation can be chosen based on the fact that values for the structural damping factor is based on experimental testing of full-scale structures. In this type of test, this factor normally is a result of the overall building–foundation damping, and not only a result of the structural damping [10]. The simple approach followed in this study, namely that a particular structural damping factor ξ represents the effective damping of the building-foundation system, is in agreement with the above-discussed recommendation, in the sense that the damping factor taken in this study for the building–foundation system would be a lower limit of the recommended value.

2.2. Earthquake response analysis of multistorey buildings supported on flexible soils

The building–foundation system under study is shown in Figure 3. The building has N floor masses, a total height equal to H, and a constant interstorey height equal to h_0. The base excitation of the system is described by the parameters v_g, v_0, and θ (Figure 4), which were also used for defining the base excitation of the single-storey structure on soft soil previously described. The parameters ω_h^e and ω_m^e are characteristic frequencies which depend on the translational and rotational stiffness of the soil–foundation system, respectively, and on the mass of the superstructure. The fundamental frequency of the multistorey building in its fixed-base case condition is ω^e. Figure 4 also shows the deflected shape of the building–foundation system under study and the roof displacement relative to the base, δ, resulting from the deformation of the superstructure; δ is a key parameter in the analysis of the seismic response of multistorey buildings [11, 12].

An approximated procedure for the seismic analysis of a multistorey building supported on flexible soil is presented in Appendix A. In this procedure, the soil–foundation system is assumed to respond in the elastic range. In addition, a basic hypothesis used in this procedure is to assume

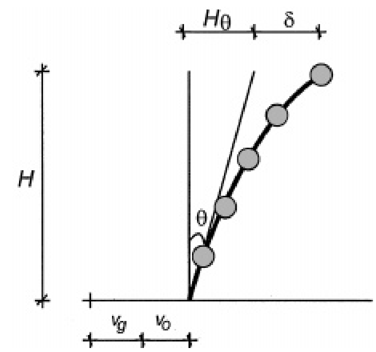

Figure 4. Components of displacements for the building–foundation system.

a constant deflected shape for the multistorey building under earthquake excitation. Such hypothesis has been used and discussed before for the fixed-base condition [11, 13, 14]. As in the case of the single-storey structure previously discussed, it is also assumed that the mass at the base and the centroidal moment of inertia of the floor masses can be neglected. In Appendix A it is shown that the use of these hypotheses in combination with the equations expressing the equilibrium of the multistorey building in coupled horizontal translation and rocking leads to the following three equations (Equations (A.11), (A.22) and (A.24) of Appendix A):

$$\ddot{\delta} + \gamma\ddot{v}_{\mathrm{g}} + \gamma\ddot{v}_0 + H^*\ddot{\theta} + 2\xi^*\omega^*\dot{\delta} + \frac{R^*}{M^*} = 0 \tag{5}$$

$$\ddot{\delta} + \gamma\ddot{v}_{\mathrm{g}} + \gamma\ddot{v}_0 + H^*\ddot{\theta} + \omega_m^{e^2}\gamma v_0 = 0 \tag{6}$$

$$\ddot{\delta} + \gamma\ddot{v}_{\mathrm{g}} + \gamma\ddot{v}_0 + H^*\ddot{\theta} + \omega_m^{e^2} H^*\theta = 0 \tag{7}$$

Equations (5)–(7) also can be viewed as the equations of motion of system Q^*, which is shown in Figure 5. An important advantage of using this simple system is that its lateral displacement with respect to the ground resulting from the deformation of the superstructure is also δ. The inspection of Equations (5)–(7) shows that the horizontal base translation of the system Q^* is that of the building-foundation system amplified by the factor γ (Figure 6), which is defined in Appendix A. The parameters ξ^* and ω^* are, respectively, the fraction of critical structural damping of system Q^* and the circular frequency of this system in the fixed-base case. The parameters R^*, M^* and H^* are the resistance function, mass, and height system Q^*, and they are defined in Appendix A. In addition, for arriving at Equations (5)–(7), it has been shown (Appendix A) that the characteristic frequencies in system Q^*, ω_h^* and ω_m^*, are equal to ω_h^e and ω_m^e, respectively.

The governing equations of motion of system Q^* correspond to those of a SDOF system. Although these equations can be considered to be well established, some of them are rederived in

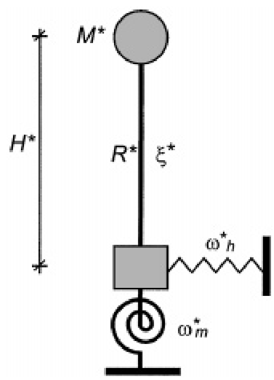

Figure 5. System Q^*.

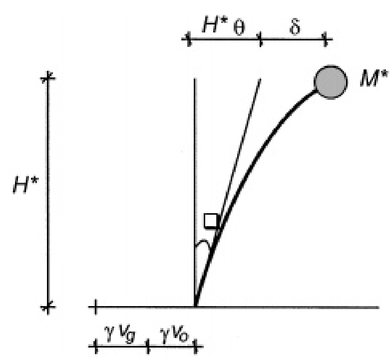

Figure 6. Components of displacements for system Q^*.

Appendix A in order to relate the characteristic parameters of system Q^* to those of the multistorey building and of the SDOF system shown in Figure 1. Also, some equations rederived in Appendix A are later used in Appendix B for evaluating inelastic strain energies in both Q^* system and the multistorey building.

Assuming the hypothesis of constant deflected shape in a multistorey building with a fixed base, during both its linear and non-linear response, and considering that the parameters μ, ω, and ξ in the single-storey structure for the fixed-base condition are equal to the corresponding parameters in system Q^*, the following expression is obtained [13, 14]:

$$\delta = \gamma u \tag{8}$$

In order to relate response parameters of the system Q^* on flexible soil and those of the single-storey system previously discussed, the effective height of the 'equivalent' single-storey system, h, is related to H^* by the following expression [15]:

$$H^* = \gamma h \tag{9}$$

Using Equation (9) and assuming several hypotheses listed in the following, it is shown in Appendix A that if u is a solution of the coupled Equations (1)–(3), it follows that Equation (8) is a solution of coupled Equations (5)–(7). An assumed hypothesis for that demonstration is that the fixed-base frequency ω^*, the fraction of critical damping ξ^*, and displacement ductility ratio μ^*, of system Q^* are equal, respectively, to the parameters ω, ξ, and μ of the single-storey structure. In addition, the characteristic frequencies ω_h^e and ω_m^e in the system Q^* are assumed equal, respectively, to the characteristic frequencies ω_h and ω_m in the single-storey structure.

2.3. Basis of the parametric study

Analyses of strong-motion and low-amplitude test data are commonly performed for investigating the dynamic characteristics of buildings. In this study, the fundamental frequency of the building–foundation system, $\bar{\omega}^e$, is related to the fixed-base fundamental frequency of the superstructure, ω^e, through the equation

$$\frac{1}{\bar{\omega}^{e^2}} = \frac{1}{\omega^{e^2}} + \frac{1}{\omega_h^{e^2}} + \frac{1}{\omega_m^{e^2}} \tag{10}$$

Equation (10) has been adopted by the ATC3-06 [4] and also by the Mexico City Building Code [16].

In order to conduct a parametric study using Equation (10) it is necessary: (1) to assume that the SSI frequency $\bar{\omega}^*$ and the fixed-base frequency ω^* in system Q^* are equal, respectively, to the frequencies $\bar{\omega}^e$ and ω^e; and (2) to use representative values of the parameters involved. Typical values for this parametric study were based on ambient vibration test data on typical buildings on soft soil in Mexico City [17], as well as on analytical studies [18]. Based on these studies, the selected values for the ratios ω_m^e/ω_h^e and $\omega^e/\bar{\omega}^e$ were 0.5 and 1.3, respectively. Since this set of values can be considered typical of medium-rise buildings in Mexico City, the conclusions from this study should be applied only to this type of buildings.

3. NUMERICAL RESULTS

Lateral strength, structural displacement and hysteretic energy demands were obtained for the parametric study conducted in this research. These results were obtained based on the solution of Equations (1)–(3), which was performed using the Drain-2DX program [19], and the EW component of the 1985 SCT, Mexico City earthquake record. The acceleration, velocity and displacement traces of this record are shown in Figure 7.

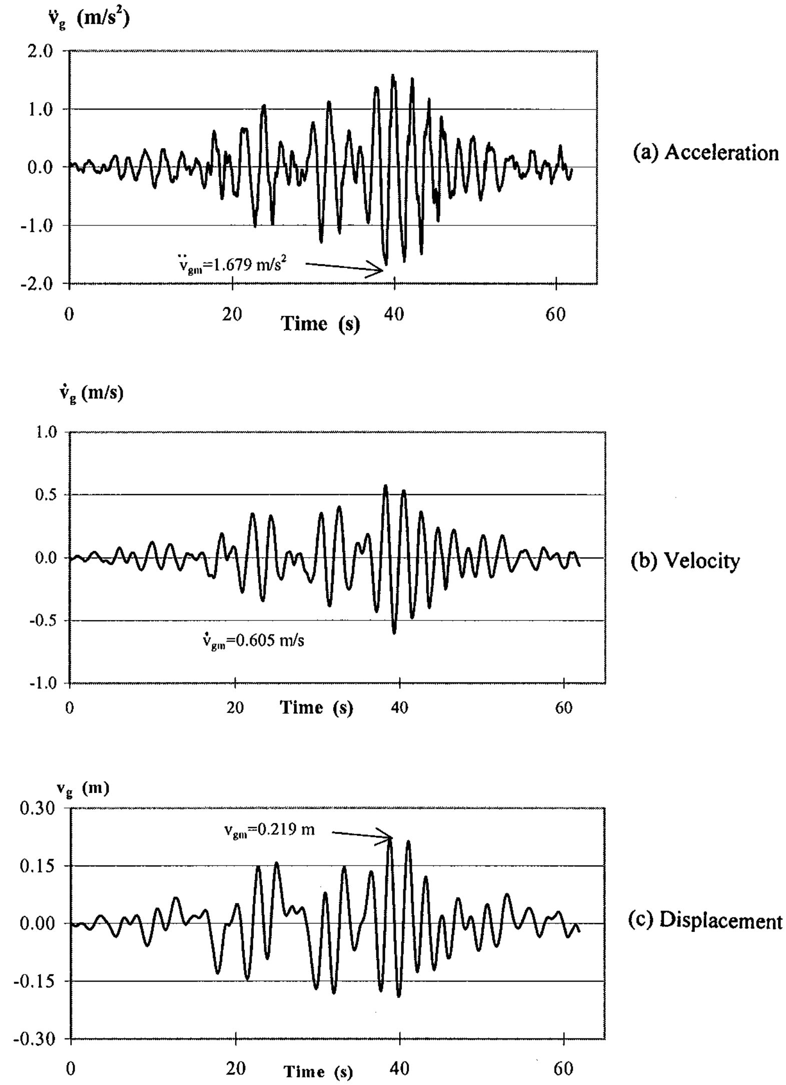

Figure 7. Acceleration, velocity and displacement traces for the 1985 SCT, Mexico City Earthquake record.

3.1. Lateral strength demands

Figure 8 shows strength demands as a function of the fundamental period of the SSI system, $\bar{T}$, for the single-storey structure in the fixed-base case and as a SSI system. These demands are conveniently non-dimensionalized in terms of parameter η, equal to the system's yield resistance per unit mass, r_y/m, divided by the peak ground acceleration. In these results, displacement ductility demands, μ, were assumed to be equal to 1, 2, 4 and 8; ξ was taken equal to 0.05. The

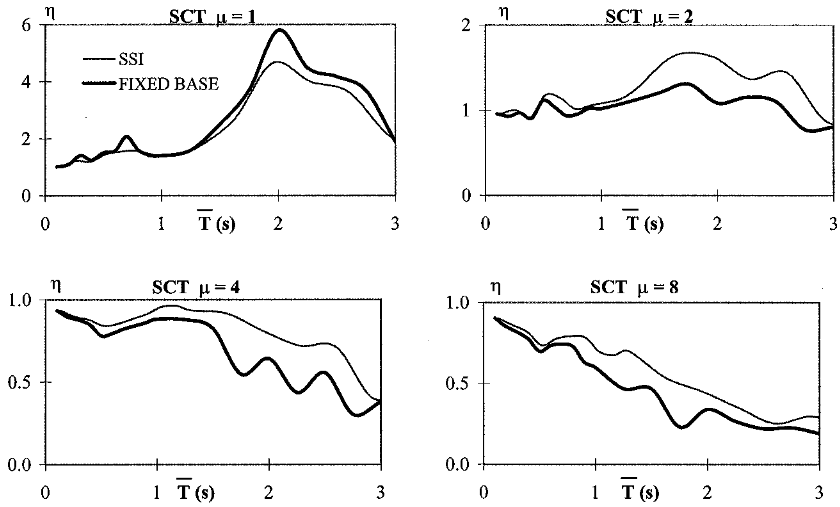

Figure 8. Strength demands, η, for single-storey structures on rigid and flexible foundations. SCT record; $\mu = 1$, 2, 4 and 8; $\xi = 0.05$. ($\bar{T}$ = SSI period).

results show that in a wide period range of inelastic cases, the strength demands for the fixed-base case using the SSI period, $\bar{T}$, are smaller than the strength demands corresponding to the flexible-base case. On the contrary, for the elastic case, the fixed-base case with the SSI period leads to some higher strength demands than those obtained for the flexible-base case. Since according to Equation (A.17) of Appendix A, the parameter r/m is directly proportional to the restoring force R^*/M^* in system Q^*, similar trends to those described for the parameter r/m are expected for R^*/M^*.

3.2. Structural displacement demands

Figure 9 show structural displacement demands for the single-storey structure responding to the SCT record in the fixed-base case and as a SSI system. The structural displacement demands, u, shown in Figure 9 are normalized with respect to the maximum ground displacement, v_{gm}. As in the case analysed in Figure 8, the values assumed for μ were equal to 1,2,4, and 8, and ξ was also taken equal to 0.05. The abscissa of Figure 9 represents also the period of the SSI system. The results show that in a wide range of periods, the fixed-base case gives a conservative estimation of the relative displacement demands in SSI systems. Nevertheless, the results show that for most of the inelastic cases the differences of results using both systems are not significant, which suggests that inelastic relative displacement demands in simple SSI systems can be evaluated by using the fixed-base case along with the amplified period of the SSI system. Similar results have also been obtained by other researchers [18].

The results shown in Figure 9 were also expressed in terms of the fixed-base natural period of the system [20]. According to these results, although in some cases of non-linear response the

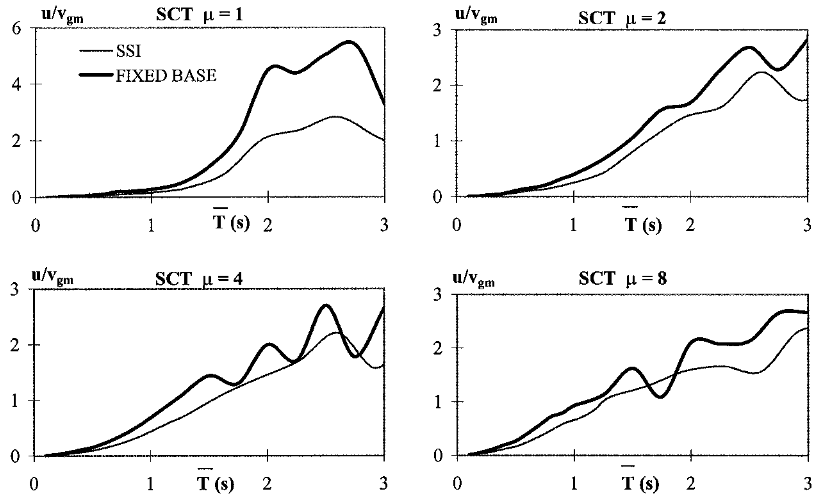

Figure 9. Displacement demands for single-storey structures on rigid and flexible foundations. SCT record; $\mu = 1$, 2, 4 and 8; $\xi = 0.05$. ($\bar{T}$ = SSI period).

interaction effect may lead to larger displacements than would occur if the base were rigid, in general the interaction effect is more important for a linear system than for a nonlinear system. Similar results have also been obtained by Bielak [1] from analysis of the steady-state harmonic response of simple non-linear structures. These results also confirm earlier findings by Veletsos and Verbic [21], who analysed the interaction effect on simple linear and nonlinear structures subjected to the 1940 El Centro record and a half-cycle displacement pulse.

For a uniform frame, using results of Figure 9 and based on Equation (8), the roof drift relative to the base, δ, resulting from the deformation of the superstructure can be evaluated. Then, the roof drift ratio, D_{rm}, defined as the ratio of the roof drift to the height of the roof above the base, H, is defined as

$$D_{rm} = \frac{\delta}{H} \tag{11}$$

Approximated procedures have been proposed for relating maximum interstorey drift ratio, d_{rm}, and the roof drift ratio D_{rm} [11, 12], which suggests the importance of the latter in assessing seismic damage in uniform structures.

Based on the above discussion, results of Figure 9 indicate that displacement analysis of a multistorey building on flexible soil can be performed considering the fixed-base case along with the amplified period of the SSI system.

3.3. Hysteretic energy demands

After solving Equations (1)–(3), hysteretic energy spectra per unit mass, E_H, for the single-storey structure considering the fixed-base and flexible-base cases were evaluated in this study. In order

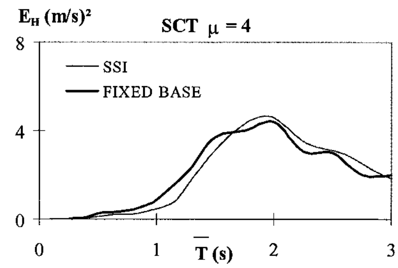

Figure 10. Hysteretic energy demands per unit mass, E_{H}, for single-storey structures on rigid and flexible foundations. SCT record; $\mu = 4$; $\xi = 0.05$ ($\bar{T}$ = SSI period).

to construct these spectra, E_{H} is evaluated at the end of the ground motion and plotted as a function of period, damping and displacement ductility ratio demands. Results are shown in Figure 10 for the case $\xi = 0.05$ and $\mu = 4$. As in the case of Figures 8 and 9, the abscissa of Figure 10 represents the period of the SSI system. Results for other values of μ are not shown here since the E_{H} spectra are proportional to those of a damage parameter discussed later. The results show that in a wide range of periods, a reasonable estimation of E_{H} can also be obtained using hysteretic energy spectra for the fixed-base case and the amplified period of the SSI system. However, in low-ductility structures, such as the case of μ equal to 2, and for amplified periods smaller than the dominant period of the soil, the proposed procedure could lead to a very conservative estimation of E_{H} [22].

In Appendix B it is shown that by using several simplified hypotheses, an approximate evaluation of the hysteretic energy dissipated by a multistorey building resting on flexible soil, E_{H}^{e}, can be done using

$$E_{\mathrm{H}}^{e} = M_{\mathrm{T}} E_{\mathrm{H}} \tag{12}$$

in Equation (12) M_{T} is the total mass of the superstructure and E_{H} is the hysteretic energy dissipated by the single-storey structure supported on flexible soil.

4. EVALUATION OF A SEISMIC DAMAGE PARAMETER FOR BUILDINGS

A seismic damage parameter proposed in the literature for assessing seismic damage in multistorey buildings supported on rigid soils [14] is briefly described in the following. The proposed parameter, I_{D}, is defined as

$$I_{\mathrm{D}} = \frac{E_{\mathrm{H}}^{*}}{E_{\lambda}^{*}} \tag{13}$$

where E^*_{H} and E^*_{λ} are the hysteretic energy and elastic strain energy, respectively, corresponding to system Q^* in the fixed-base case. E^*_{H} can be evaluated from

$$E^*_{\mathrm{H}} = \gamma^2 E_{\mathrm{H}} \tag{14}$$

and E^*_{λ} is given by the following expression:

$$E^*_{\lambda} = (2\pi\lambda h_0 D_{\mathrm{rd}})^2 \tag{15}$$

in Equation (15) λ is a parameter that allows an approximate evaluation of the fundamental period in a regular structure based on the following expression:

$$T = \frac{N}{\lambda} \tag{16}$$

In the case of regular frames on rigid base, a value of 7 can be used for λ, which results from the commonly used value of 10 for estimating fundamental periods of vibrations in regular frames subjected to small amplitude vibrations and from considering a reduction of about 50 per cent in lateral stiffness during earthquakes.

The parameter D_{rd} is an acceptable roof drift ratio in a building when responding to an earthquake, associated to the roof drift δ_{d}, that is [14]:

$$D_{\mathrm{rd}} = \frac{\delta_{\mathrm{d}}}{H} \tag{17}$$

In Appendix B it is shown that E^*_{H} in system Q^* supported on flexible soil can be evaluated using Equation (14), where E_{H} is the hysteretic energy dissipated by the single-storey structure on flexible soil. Considering this finding and combining Equations (13)–(15) we obtain

$$I_{\mathrm{D}} = \frac{\gamma^2 E_{\mathrm{H}}}{(2\pi\lambda h_0 D_{\mathrm{rd}})^2} \tag{18}$$

According to Equation (18), for a specific structural system and for a specific value of D_{rd}, I_{D} can be considered in direct proportion to E_{H}, which suggests the importance of this parameter for assessing seismic damage in buildings supported on flexible soils. The importance of an appropiate structural system for reducing seismic damage can also be identified from inspecting Equation (18), since according to this equation the parameters λ and D_{rd} are relevant in the evaluation of I_{D}.

Following the procedure above described for evaluating I_{D}, and after solving Equations (1)–(3), Figure 11 shows results for the cases of regular frames supported on rigid and flexible soils. In this evaluation, γ and λ were taken equal to 1.5 and 7, respectively, and h_0 and D_{rd} were assumed to be equal to 2.7 m and 0.01, respectively. Since according to the previous discussion, for a specific structural system and for some specific parameters values, E_{H} can be considered in direct proportion to I_{D}, the relative differences of I_{D} values corresponding to the fixed-base and flexible-base cases are the same as those previously discussed for E_{H} in both cases of supporting soils. The

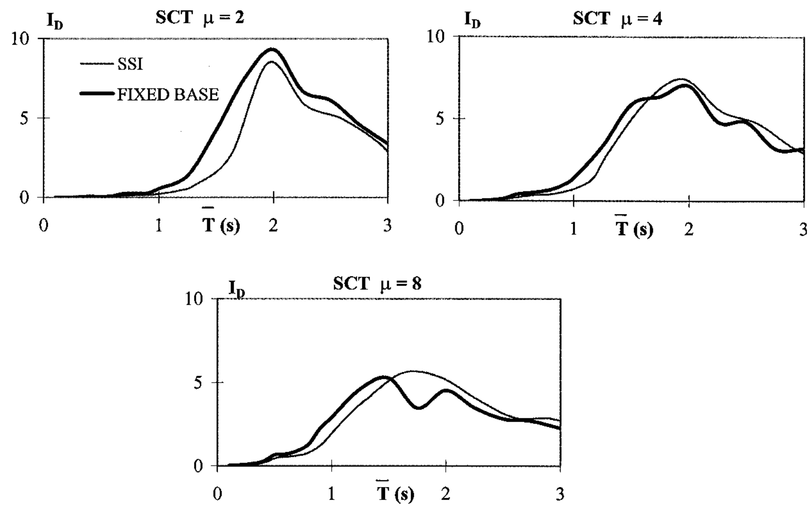

Figure 11. Seismic damage parameter, I_D, for multistorey regular frames on rigid and flexible foundations. SCT record; $\mu = 2$, 4 and 8; $\xi = 0.05$. ($\bar{T}$ = SSI period).

results in Figure 11 show that using the I_D spectra for the rigid-base case along with the amplified period of the SSI system, a conservative estimation of I_D for buildings on flexible soils can be obtained in most cases. However, in other cases this procedure might yield some unconservative results.

The results found in this study for the parameters δ and I_D for the fixed-base and flexible-base cases, show that for most cases of multistorey buildings supported on flexible soils, a simplified seismic damage assessment can be performed by considering the fixed-base case of associated single-storey structures along with the amplified period of the corresponding SSI systems. However, caution should be taken when extrapolating the proposed procedure for assessing seismic damage considering pounding in buildings. In this type of seismic damage, not only δ should be considered, but also the contribution to the roof drift from the base rotation and translation, which has not been considered in this study.

5. CONCLUSIONS

A parametric study of the SSI effects on the inelastic seismic response of building–foundation systems using a simple approach has been presented. The results indicate that in most cases of inelastic response, SSI effects are not very significant, which is in agreement with earlier findings by other authors. The results for both fixed-base and SSI systems show that for a wide range of periods in multistorey buildings supported on flexible soils, such as those typical of Mexico City, roof drifts and hysteretic energy demands can be obtained using the results for the fixed-base case and the amplified period of the SSI system. The results also show that a similar procedure can be followed for assessing seismic damage in multistorey buildings supported on flexible soils.

Caution should be taken when extrapolating the proposed procedure for assessing seismic damage considering pounding in SSI systems, since this effect has not been considered in this study.

APPENDIX A. SEISMIC ANALYSIS OF MULTISTOREY BUILDINGS ON FLEXIBLE SOILS

In the following, the soil–foundation system is assumed to respond in the elastic range. As discussed in the paper, an effective damping of the interacting system is also assumed. The multistorey building under study is shown in Figure 3.

A.1. Equation of motion considering horizontal forces in the superstructure

The equation of motion considering the horizontal forces in the superstructure of the system shown in Figure 4 is

$$[M][\{\ddot{U}\} + \{1\}\ddot{v}_g + \{1\}\ddot{v}_0 + \{H\}\ddot{\theta}] + [C]\{\dot{U}\} + \{R\} = 0 \tag{A.1}$$

where $[M]$ is the mass matrix for the superstructure, $\{U\}$ the vector of displacements relative to the base, $\{1\}$ vector of N elements equal to 1, $\{H\}$ the vector in which the jth term is the height of the jth level with respect to the base, $[C]$ the damping matrix, $\{R\}$ the vector corresponding to the restoring forces in the superstructure.

A constant deflected shape $\{\phi\}$ is assumed, which leads to:

$$\{U\} = \{\phi\}\delta \tag{A.2}$$

where δ is the roof displacement relative to the base resulting from the deformation of the superstructure.

It is convenient to introduce the following notation:

$$M^* = \{\phi\}^{\mathrm{T}}[M]\{\phi\} \tag{A.3}$$

$$L^* = \{\phi\}^{\mathrm{T}}[M]\{1\} \tag{A.4}$$

$$J_1^* = \{\phi\}^{\mathrm{T}}[M]\{H\} \tag{A.5}$$

$$C^* = \{\phi\}^{\mathrm{T}}[C]\{\phi\} \tag{A.6}$$

$$R^* = \{\phi\}^{\mathrm{T}}\{R\} \tag{A.7}$$

$$\gamma = L^*/M^* \tag{A.8}$$

$$H^* = J_1^*/M^* \tag{A.9}$$

$$C^*/M^* = 2\xi^*\omega^* \tag{A.10}$$

Substitution from Equation (A.2) Equation (A.1) and premultiplication by $\{\phi\}^{\mathrm{T}}$, and considering the notation given in Equations (A.3)–(A.10) leads to

$$\ddot{\delta} + \gamma\ddot{v}_{\mathrm{g}} + \gamma\ddot{v}_0 + H^*\ddot{\theta} + 2\xi^*\omega^*\dot{\delta} + \frac{R^*}{M^*} = 0 \tag{A.11}$$

Equation (A.11) can be viewed as an equation of motion of system Q^*, which is shown in Figure 5. When considering the fixed-base case, this system behaves as a SDOF system. Inspection of Equation (A.11) shows that the horizontal base translation of system Q^* is that of the building–foundation system amplified by the factor γ. The parameter H^* can be considered as the height of system Q^*. The parameters C^*, ω^* and ξ^* correspond to the damping coefficient, circular frequency, and fraction of critical damping for the superstructure of the system Q^* on a rigid base. The parameters R^* and M^* are the restoring force and mass, respectively, corresponding to the system Q^*.

Solution of Equation (A.11) in the range $\delta < \delta_y$

The restoring force terms of Equations (A.11) and (1) can be written, respectively, as

$$R^*/M^* = \omega^{*2}\delta \tag{A.12}$$

$$r/m = \omega^2 u \tag{A.13}$$

Substitution from Equation (A.12) into equation (A.11) leads to

$$\ddot{\delta} + \gamma\ddot{v}_{\mathrm{g}} + \gamma\ddot{v}_0 + H^*\ddot{\theta} + 2\xi^*\omega^*\dot{\delta} + \omega^{*^2}\delta = 0 \tag{A.14}$$

The effective height of the "equivalent" single-storey system, h, whose response is related to that of the multistorey system under consideration, can be evaluated from the following expression [15]:

$$H^* = \gamma h \tag{A.15}$$

Substitution from Equation (A.15) into equation (A.14) and comparing the resulting equation with the combination of Equations (1) and (A.13), shows that when ω^* and ξ^* in system Q^* are equal, respectively, to ω and ξ in single-storey structure, then if u is the solution of Equation (1), γu would be the solution of (A.14), that is

$$\delta = \gamma u \tag{A.16}$$

Combining Equations (A.12), (A.13) and (A.16), when $\omega = \omega^*$, yields

$$\frac{R^*}{M^*} = \gamma\frac{r}{m} \tag{A.17}$$

From Equation (A.16) we can write

$$\delta_y = \gamma u_y \tag{A.18}$$

Equations (A.17) and (A.18) are used in the following.

Solution of the Equation (A.11) in the inelastic range

For solving Equation (A.11) in the inelastic range, it is assumed that the global displacement ductility ratio of the building and the displacement ductility ratio of the single-storey structure are equal, i.e.

$$\frac{\delta}{\delta_y} = \frac{u}{u_y} \tag{A.19}$$

Equation (A.16) can also be obtained by combining Equations (A.18) and (A.19). Therefore, if u is the solution of the non-linear Equation (1), γu would be the solution of Equation (A.11) in the inelastic range.

Substituting Equation (A.17) into Equation (A.11) yields

$$\ddot{\delta} + \gamma \ddot{v}_g + \gamma \ddot{v}_0 + H^* \ddot{\theta} + 2\xi^* \omega^* \dot{\delta} + \gamma \frac{r}{m} = 0 \tag{A.20}$$

Inspection of the equation above and of Equation (1) shows that when damping and frequency parameters in both equations are equal, and when Equation (A.15) is considered, the solution of Equation (A.20) is given by Equation (A.16), which proves that Equation (A.17) is also valid for the inelastic range and that Equation (A.20) is another form of Equation (A.11).

A.2. Equation of motion considering horizontal forces including the foundation

The equation of motion considering the horizontal forces including the foundation of the system shown in Figure 3 is

$$\{1\}^{\mathrm{T}}[M][\{\ddot{U}\} + \{1\}\ddot{v}_g + \{1\}\ddot{v}_0 + \{H\}\ddot{\theta}] + m_0^e(\ddot{v}_g + \ddot{v}_0) + k_h^e v_0 = 0 \tag{A.21}$$

where k_h^e and m_0^e are, respectively, the horizontal stiffness and mass for the soil–foundation system.

Assuming a constant deflected shape, using approximations introduced by Luco et al. [23], and neglecting the mass at the base [1], Equation (A.21) yields

$$\ddot{\delta}\gamma \ddot{v}_g + \gamma \ddot{v}_0 + H^* \ddot{\theta} + \omega_h^{e^2} \gamma v_0 = 0 \tag{A.22}$$

where ω_h^e is the characteristic frequency for the soil–foundation of the multistorey building.

Considering Equation (A.15), assuming ω_h^e equal to ω_h, and combining (A.16) and (A.22) we get Equation (2), which shows that if u is the solution of Equation (2), γu would be the solution of Equation (A.22).

A.3. Equation of motion considering equilibrium of moments

The equation of motion considering the equilibrium of moments of the system shown in Figure 3 yields

$$\{H\}^{\mathrm{T}}[M][\{\ddot{U}\} + \{1\}\ddot{v}_{\mathrm{g}} + \{1\}\ddot{v}_0 + \{H\}\ddot{\theta}] + I^e_{m0}\ddot{\theta} + k^e_m\theta = 0 \tag{A.23}$$

where k^e_m and I^e_{m0} are, respectively, the rotational stiffness and inertia for the soil–foundation system.

Assuming a constant deflected shape, using approximations introduced by Luco et al. [23], and neglecting the centroidal moment of inertia of the top mass of the system Q^* [1], Equation (A.23) yields

$$\ddot{\delta} + \gamma\ddot{v}_{\mathrm{g}} + \gamma\ddot{v}_0 + H^*\ddot{\theta} + \omega^{e^2}_m H^*\theta = 0 \tag{A.24}$$

where ω^e_m is the characteristic frequency for the soil–foundation system of the multistorey building.

Substitution of Equation (A.15) into Equation (A.24) shows that when ω^e_m and ω_m are equal, if u is the solution of Equation (3), then γu would be the solution of Equation (A.24).

APPENDIX B. EARTHQUAKE ENERGY ANALYSIS OF MULTISTOREY BUILDINGS SUPPORTED ON FLEXIBLE SOILS

Using a procedure developed by Rodriguez [14] for buildings on rigid foundations, in the following it is shown that for the structural systems on flexible soils considered in this study, the corresponding strain energies can be related. The systems considered are a single-storey structure, the system Q^*, and the associated multistorey building.

For a single-storey structure on a rigid foundation, the sum of the hysteretic energy (E_{H}) plus the elastic strain energy per unit mass (E_{S}) is given by [24]:

$$E_{\mathrm{H}} + E_{\mathrm{S}} = \int \frac{r}{m}\,\mathrm{d}u \tag{B.1}$$

Integration of the differential equation of motion of system Q^* supported on flexible soil, Equation (A.20), with respect to the displacement δ leads to

$$\int \ddot{\delta}\,\mathrm{d}\delta + \int \gamma\ddot{v}_{\mathrm{g}}\,\mathrm{d}\delta + \int \gamma\ddot{v}_0\,\mathrm{d}\delta + \int H^*\ddot{\theta}\,\mathrm{d}\delta + 2\xi^*\omega^*\int \dot{\delta}\,\mathrm{d}\delta + \int \gamma\frac{r}{m}\,\mathrm{d}\delta = 0 \tag{B.2}$$

Substitution from Equation (A.16) into Equation (B.2) yields

$$\gamma^2\int \ddot{u}\,\mathrm{d}u + \gamma^2\int \ddot{v}_{\mathrm{g}}\,\mathrm{d}u + \gamma^2\int \ddot{v}_0\,\mathrm{d}u + \gamma^2\int h\ddot{\theta}\,\mathrm{d}u + 2\xi^*\omega^*\gamma^2\int \dot{u}\,\mathrm{d}u + \gamma^2\int \frac{r}{m}\,\mathrm{d}u = 0 \tag{B.3}$$

The last term of Equation (B.3) represents the hysteretic energy per unit mass of system Q^* supported on flexible soil (E^*_{H}) plus the elastic strain energy per unit mass of this system (E^*_{S}), that is

$$E^*_{\mathrm{H}} + E^*_{\mathrm{S}} = \gamma^2 \int \frac{r}{m}\,\mathrm{d}u \tag{B.4}$$

Assuming that Equation (B.1) is also valid for the case of a single-storey structure on flexible soil, the combination of Equation (B.1) and (B.4) leads to

$$E^*_{\mathrm{H}} + E^*_{\mathrm{S}} = \gamma^2 (E_{\mathrm{H}} + E_{\mathrm{S}}) \tag{B.5}$$

Using the definition of hysteretic energy, from Equation (B.5) we can write

$$E^*_{\mathrm{H}} = \gamma^2 E_{\mathrm{H}} \tag{B.6}$$

Since Equation (A.20) is another form of Equation (A.11), the hysteretic energy dissipated by a multi-storey building on flexible soil (E^e_{H}) can be related to E^*_{H} with the following expression:

$$E^*_{\mathrm{H}} = \frac{E^e_{\mathrm{H}}}{M^*} \tag{B.7}$$

The following approximation is introduced [23]:

$$M_{\mathrm{T}} \cong \gamma^2 M^* \tag{B.8}$$

where M_{T} is the total mass of the superstructure of the multi-storey building.

The combination of Equations (B.6)–(B.8) leads to

$$E^e_{\mathrm{H}} = M_{\mathrm{T}} E_{\mathrm{H}} \tag{B.9}$$

The expression above shows that with the assumed hypotheses, an approximated evaluation of the hysteretic energy dissipated by a multistorey regular building on flexible soil can be performed using the hysteretic energy dissipated by an associated single-storey structure.

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